The DynGenPar Algorithm on an Example

and a comparison with existing approaches

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Example Grammar

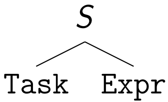
- simple grammar for unconstrained polynomial optimization problems
- $N = \{S, \texttt{Task}, \texttt{Expr}, \texttt{Term}, \texttt{Factor}\}$
- $T = \{\min, \max, +, *, x, \texttt{NUMBER}\}$
- $S \to \texttt{Task Expr}$
- $\texttt{Task} \to \min \mid \max$
- $\texttt{Expr} \to \texttt{Expr} + \texttt{Term} \mid \texttt{Term}$
- $\texttt{Term} \to \texttt{Term} * \texttt{Factor} \mid \texttt{Factor}$
- $\texttt{Factor} \to x \mid \texttt{NUMBER}$
- Example sentence: $\min x * x$

Example Top-Down Parsing (1/3)

- input: $. \min x * x$ (dot ... cursor position)
- current sentence form: $. \ S$
- current parse tree: S
- apply $S \rightarrow \texttt{Task Expr}$

Example Top-Down Parsing (2/3)

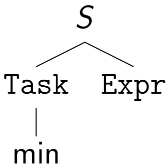
- input: $.\min x * x$ (dot . . . cursor position)
- current sentence form: . `Task Expr`
- current parse tree:



- possible choices:
  1. `Task` $\longrightarrow$ min
  2. `Task` $\longrightarrow$ max
- only rule 1 matches input token min
- apply `Task` $\longrightarrow$ min

Example Top-Down Parsing (3/3)

- min is already a terminal, accept it
- input: min . $x * x$ (dot . . . cursor position)
- current sentence form: min . Expr
- current parse tree:



- possible choices:
  1. Expr $\rightarrow$ Expr $+$ Term
  2. Expr $\rightarrow$ Term
- Problem: Which rule to apply? (Left recursion!)

Example Bottom-Up Parsing (1/5)

- current sentence form: $.\,\mathrm{min}\, x * x$ (dot . . . cursor position)
- no input read in yet
  - thus cannot reduce anything
- only option: shift a token

Example Bottom-Up Parsing (2/5)

- current sentence form: min . $x * x$
- left of the cursor: min
  - right hand side of `Task` $\rightarrow$ min
- no further shift makes sense
  - $\nexists$ rule $X \rightarrow$ min . . .
- thus reduce `Task` $\rightarrow$ min

Example Bottom-Up Parsing (3/5)

- current sentence form: `Task` . $x * x$
- left of the cursor: `Task`
  - not the right hand side of a rule
  - start of $S \longrightarrow$ `Task Expr`
  - thus need more tokens
- perform a shift step

Example Bottom-Up Parsing (4/5)

- current sentence form: `Task` x . $*$ x
- reduce step: `Factor` $\longrightarrow x$
- current sentence form: `Task Factor` . $*$ x
- reduce step: `Term` $\longrightarrow$ `Factor`
- current sentence form: `Task Term` . $*$ x
- what not?
  - reduce `Expr` $\longrightarrow$ `Term`?
  - or shift step?
  - for that, consider the token after the cursor
    - 1 Token *Lookahead* $\rightsquigarrow$ *LR(**1**) method*
  - with lookahead $*$, only a shift makes sense

Example Bottom-Up Parsing (5/5)

- current sentence form: Task Term $*$. x
- shift step
- current sentence form: Task Term $*$ x .
- reduce step: Factor $\rightarrow$ x
- current sentence form: Task Term $*$ Factor .
- reduce step: Term $\rightarrow$ Term $*$ Factor
- current sentence form: Task Term .
- lookahead empty, no more shift possible
  - reduce step: Expr $\rightarrow$ Term
- current sentence form: Task Expr .
- lookahead empty, no more shift possible
  - reduce step: $S \rightarrow$ Task Expr
- we obtain the sentence form: S . $\Rightarrow$ done!

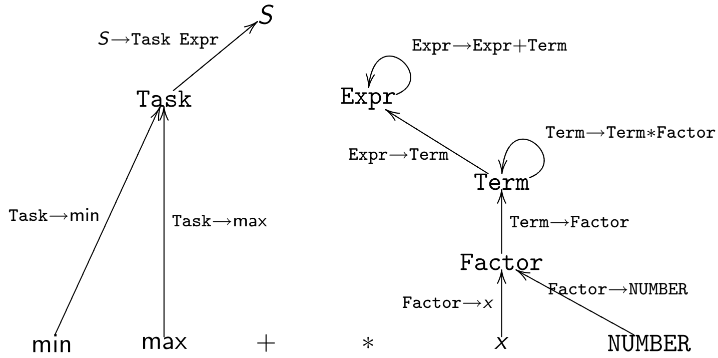
Initial Graph

- replaces precompiled tables
  - dynamically extensible for new rules
- directed, labeled multigraph on $\Gamma = N \cup T$
- tokens T are sources
- edge from symbol $s \in \Gamma$ to category $n \in N$ iff
  - $\exists$ rule $n \to n_1 \; n_2 \; \ldots \; n_k \; s \; \ldots$ with $n_i \in N_0 \; \forall i$
    - where $N_0 \subseteq N$ the set of all nonterminals from which ε can be derived
  - label of the edge
    - that rule
    - number k of n_i set to ε
  - more than one possible label … multi-edge

Example Initial Graph (1/2)

- Example grammar
  - $N = \{S, \texttt{Task}, \texttt{Expr}, \texttt{Term}, \texttt{Factor}\}$
  - $T = \{\text{min}, \text{max}, +, *, x, \texttt{NUMBER}\}$
  - $S \to \texttt{Task Expr}$
  - $\texttt{Task} \to \text{min} \mid \text{max}$
  - $\texttt{Expr} \to \texttt{Expr} + \texttt{Term} \mid \texttt{Term}$
  - $\texttt{Term} \to \texttt{Term} * \texttt{Factor} \mid \texttt{Factor}$
  - $\texttt{Factor} \to x \mid \texttt{NUMBER}$
- $N_0 = \emptyset$
  - because there is no rule $n \to \varepsilon$ in the grammar
  - thus consider only rules of the form $n \to s \ \ldots$
  - k (number of skipped ε categories) $= 0$ everywhere

Example Initial Graph (2/2)

- we obtain the graph
  - 

Neighborhoods

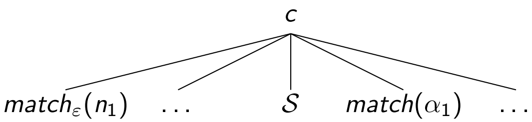
- Let $s \in \Gamma = N \cup T$ (*symbol*), $z \in N$ (*target*)
- *Neighborhood* $\mathcal{N}(s, z)$ ...
  - Edges from s to a category c where
    - z reachable from c
- in the example
  - $\mathcal{N}(\mathrm{min}, S) = \{\mathtt{Task} \rightarrow \mathrm{min}\}$
  - $\mathcal{N}(x, S) = \emptyset$
  - $\mathcal{N}(x, \mathtt{Expr}) = \{\mathtt{Factor} \rightarrow x\}$
  - $\mathcal{N}(\mathtt{Term}, \mathtt{Expr}) = \{\mathtt{Expr} \rightarrow \mathtt{Term}, \mathtt{Term} \rightarrow \mathtt{Term} * \mathtt{Factor}\}$
- computed by graph walk
- can be cached
  - but must be recomputed if the grammar changes

Operations

- $match_\varepsilon(n), n \in N_0$ ... derive ε from n
  - top-down
  - ignore recursion (would produce ∞ly many parse trees)
- *shift* ... read in the next token
- $reduce(s, z), s \in \Gamma, z \in N$... reduce s to z
  - different from LR *reduce*!
  - already reduce after the first symbol
  - *reduce* must complete the match
- $match(s), s \in \Gamma = N \cup T$
  - if $s \in N_0$: $match_\varepsilon(s)$, remember result
  - $t = shift$
  - if $s \in T$: compare s with t
  - if $s \in N$: $reduce(t, s)$

Reduce(s,z) Step

- pick a rule $c \to n_1 \; n_2 \; \ldots \; n_k \; s \; \alpha_1 \; \alpha_2 \; \ldots \; \alpha_\ell$ in $\mathcal{N}(s, z)$
- for each $n_i \in N_0$:
  - $match_\varepsilon(n_i)$
- s already recognized . . . parse tree $\mathcal{S}$
- for each $\alpha_j \in \Gamma = N \cup T$:
  - $match(\alpha_j)$ (top-down step)
- parse tree:



- if $c \neq z$: continue reducing recursively
  - $reduce(c, z)$
- (also consider $reduce(z, z)$ for left recursion)

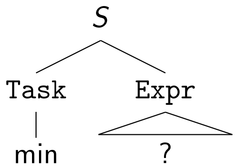
Example DynGenPar Algorithm (1/9)

- in the example: $N_0 = \emptyset \Rightarrow$ no $match_\varepsilon$ steps
- input: $.\,\mathrm{min}\, x * x$ (dot ... cursor position)
- begin with $match(S)$ (start category)
  - shift step produces min
  - next step: $reduce(\mathrm{min}, S)$
  - input now: $\mathrm{min}\,.\; x * x$

Example DynGenPar Algorithm (2/9)

- $reduce(\text{min}, S)$
  - $\mathcal{N}(\text{min}, S) = \{\texttt{Task} \rightarrow \text{min}\}$
  - thus reduce $\texttt{Task} \rightarrow \text{min}$
  - $k = 0$ (no n_i), $\ell = 0$ (no α_j)
  - thus continue with $reduce(\texttt{Task}, S)$
    - $\mathcal{N}(\texttt{Task}, S) = \{S \rightarrow \texttt{Task Expr}\}$
    - thus reduce $S \rightarrow \texttt{Task Expr}$
    - now we have an α_j: $\alpha_1 = \texttt{Expr}$
    - thus $match(\texttt{Expr})$
    - then reduce complete, because S is already the goal

Example DynGenPar Algorithm (3/9)

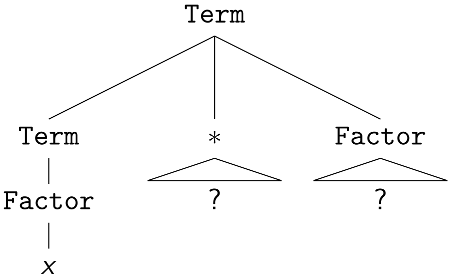
- already recognized:
  - 

- *match*(Expr)
  - shift step produces x
  - next step: $reduce(x, \texttt{Expr})$
  - input now: min x $. * x$

Example DynGenPar Algorithm (4/9)

- *reduce*$(x, \texttt{Expr})$
  - $\mathcal{N}(x, \texttt{Expr}) = \{\texttt{Factor} \to x\}$
  - continue with *reduce*$(\texttt{Factor}, \texttt{Expr})$
  - $\mathcal{N}(\texttt{Factor}, \texttt{Expr}) = \{\texttt{Term} \to \texttt{Factor}\}$
  - continue with *reduce*$(\texttt{Term}, \texttt{Expr})$
  - $\mathcal{N}(\texttt{Term}, \texttt{Expr}) = \{\texttt{Expr} \to \texttt{Term}, \texttt{Term} \to \texttt{Term} * \texttt{Factor}\}$
    - reduce-reduce conflict
    - must consider both possibilities
    - $\texttt{Expr} \to \texttt{Term}$: *reduce*$(x, \texttt{Expr})$ and thus *match*$(\texttt{Expr})$ terminates (or try reducing $\texttt{Expr} \to \texttt{Expr}+\texttt{Term} \Rightarrow$ error), thus also *reduce*(min, S) and *match*(S), but the input is not consumed yet $\Rightarrow$ error
    - thus reduce $\texttt{Term} \to \texttt{Term} * \texttt{Factor}$
    - thus *match*$(*)$ and *match*$(\texttt{Factor})$

Example DynGenPar Algorithm (5/9)

- already recognized:
  - 

- *match*$(*)$ trivial (only shift)
- input now: $\min x * . \; x$

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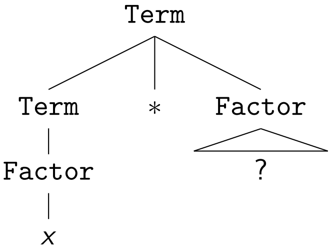
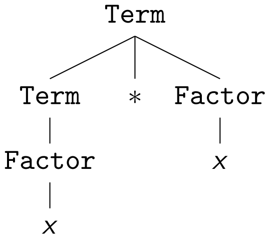
Example DynGenPar Algorithm (6/9)

- already recognized:
  - 

- *match*(Factor)
  - shift step produces x
  - next step: *reduce*(x, Factor)
  - input now consumed: min $x * x$.

Example DynGenPar Algorithm (7/9)

- $\textit{reduce}(x, \texttt{Factor})$
  - $\mathcal{N}(x, \texttt{Factor}) = \{\texttt{Factor} \to x\}$
  - so we are already done
- $\texttt{Term}$ recognized completely
  - 

- remaining open target: *Expr*
  - thus continue with another $\textit{reduce}(\texttt{Term}, \texttt{Expr})$

Example DynGenPar Algorithm (8/9)

- *reduce*$(\texttt{Term}, \texttt{Expr})$
  - $\mathcal{N}(\texttt{Term}, \texttt{Expr}) = \{\texttt{Expr} \to \texttt{Term}, \texttt{Term} \to \texttt{Term} * \texttt{Factor}\}$
    - again 2 possibilities
    - this time, $\texttt{Term} \to \texttt{Term}{*}\texttt{Factor}$ fails (out of input)
    - thus reduce $\texttt{Expr} \to \texttt{Term}$
    - continue with *reduce*$(\texttt{Expr}, \texttt{Expr})$
    - $\texttt{Expr} \to \texttt{Expr}{+}\texttt{Term}$ fails (out of input)
    - thus we are done
- and so the input has been recognized completely

Example DynGenPar Algorithm (9/9)

- we obtain the same parse tree as for LR:
  -