Dynamic Generalized Parsing and Natural Mathematical Language

Defensio

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- many more people (too many to list here).

What is Parsing?

- automatic input of text into a form understandable to a computer
- Input: **text** (sequence of characters from an **alphabet**)
- Output: **syntax tree** (tree representation of the text structure)
- often in 2 steps:
  - **lexing** (**scanning**):
    - Input: text
    - Output: sequence of **tokens**
  - **parsing** (in a strict sense):
    - Input: sequence of tokens
    - Output: syntax tree
- **scannerless parsing**: tokens = characters

Grammars

- fundamental information for parsing
- describe matching between text and syntax tree
- **grammar** = set of rules
  - variables in rules: **categories**
  - **rule**: how to generate text from a category
    - start from **start category**
    - parsing: reversed reading
- most frequent case: **context-free grammars**
  - rules of the form $\texttt{Cat} \to \alpha\ \beta\ \gamma \dots$
    - where $\alpha, \beta, \gamma, \dots$ categories or tokens
    - e.g., $\texttt{S} \to \texttt{A b}$, $\texttt{A} \to \texttt{A a}$, $\texttt{A} \to \texttt{a}$ (produces `ab`, `aab`, `aaab`, etc.)
  - formal definition in the next section

Types of Mathematical Text

- **natural language text**:
  The cost is the sum of the area of the square and half of its perimeter. Minimize the cost.
- **formulas**: $\min x^2 + 2x$
  - programming language style: `min(x^2+2*x)`
  - LaTeX style: `$\min x^2+2x$`
- **mixed**: *Minimize* $x^2 + 2x$.

Primary Target: FMathL (Formal Mathematical Language)

A **modeling language** is an artificial language for the user friendly specification of mathematical problems, with interfaces to the corresponding solvers.

FMathL is intended to be a modeling and documentation language for the working mathematician that

- is based on traditional mathematical syntax,
- allows to express arbitrary mathematics,
- decides automatically which tools to use.

FMathL Goals

- modeling language for optimization problems (short term)
- build computer-oriented library of math. knowledge
  - formalized for the computer
  - from input as informal as possible
    - textbooks and papers (e.g., arXiv papers)
- reasoning
  - checking the correctness of proofs
  - solving computational (e.g., optimization) problems
  - semantic classification and retrieval
    - search engine for mathematics
- (mostly) automatic translation of mathematical texts
- vision: MathResS –
  automatic mathematical research system

Requirements

- allow efficient incremental addition of new rules
  - without recompiling the whole grammar
  - e.g. for a mathematical definition
- more general grammars than LR(1) or LALR(1)
  - natural language is usually not LR(1)
  - parallel multiple context-free grammars (PMCFGs)
- exhaustively produce all possible parse trees
  - packed representation
- predictive parsing
  - incremental processing
  - predicting the next token
- both scanner-driven (natural math. language) and scannerless (most other parsing tasks)

Dynamic Generalized Parser

- dynamic
  - grammar not fixed in advance
  - allows adding rules at any time
    - even during parsing (mathematical definitions)
  - avoids precompiled tables
- generalized
  - general context-free grammars
  - produces all parse trees for ambiguous grammars
  - additional generalization: PMCFGs (Seki et al.)
    - Parallel Multiple Context-Free Grammars
    - interoperability with GF (Grammatical Framework)
- simplicity is important
  - formal verifiability (in the future)

Problems with Existing Approaches

- top-down parsing
  - chokes on left recursion
  - e.g. $\texttt{Expr} \to \texttt{Expr} + \texttt{Term} \mid \texttt{Term}$
  - requires grammar transformation or complex workarounds
    - fails simplicity criterion
- bottom-up parsing
  - problem: what operation (shift, reduce) to do when
  - requires parse states and stacks, lookahead
  - requires precomputed tables
    - have to recompute the whole table when the grammar changes

Definition: Context-free Grammar (CFG)

- $G = (N, T, P, S)$
- N ... (finite) set of **nonterminals** (**categories**)
- T ... (finite) set of **tokens**
  - called **alphabet** of the grammar
  - disjoint from N
- P ... (finite) set of **productions** (**rules**)
  - of the form $n \to \alpha_1 \dots \alpha_k, n \in N, \alpha_i \in N \cup T \; \forall i$
- $S \in N$ ... **start symbol** (**start category**)

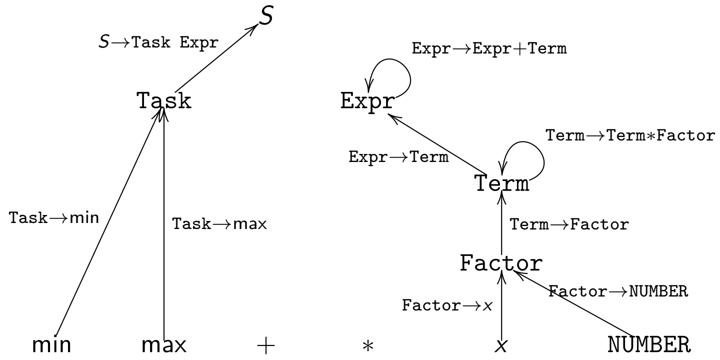
Initial Graph

- replaces precompiled tables
  - dynamically extensible for new rules
- directed, labeled multigraph on $\Gamma = N \cup T$
- tokens T are sources
- edge from symbol $s \in \Gamma$ to category $n \in N$ iff
  - $\exists$ rule $n \to n_1 \ n_2 \ \ldots \ n_k \ s \ \ldots$ with $n_i \in N_0 \ \forall i$
    - where $N_0 \subseteq N$ the set of all nonterminals from which ε can be derived
  - label of the edge
    - that rule
    - number k of n_i set to ε
  - more than one possible label . . . multi-edge

Example Initial Graph (1/2)

- Example grammar
  - $N = \{S, \texttt{Task}, \texttt{Expr}, \texttt{Term}, \texttt{Factor}\}$
  - $T = \{\text{min}, \text{max}, +, *, x, \texttt{NUMBER}\}$
  - $S \to \texttt{Task Expr}$
  - $\texttt{Task} \to \text{min} \mid \text{max}$
  - $\texttt{Expr} \to \texttt{Expr} + \texttt{Term} \mid \texttt{Term}$
  - $\texttt{Term} \to \texttt{Term} * \texttt{Factor} \mid \texttt{Factor}$
  - $\texttt{Factor} \to x \mid \texttt{NUMBER}$
- $N_0 = \emptyset$
  - because there is no rule $n \to \varepsilon$ in the grammar
  - thus consider only rules of the form $n \to s \ \ldots$
  - k (number of skipped ε categories) $= 0$ everywhere

Example Initial Graph (2/2)

- we obtain the graph
  - 

Neighborhoods

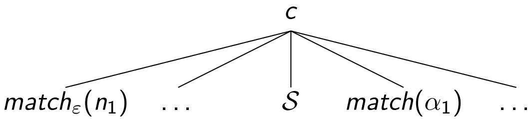
- Let $s \in \Gamma = N \cup T$ (*symbol*), $z \in N$ (*target*)
- *Neighborhood* $\mathcal{N}(s, z) \ldots$
  - Edges from s to a category c where
    - z reachable from c
- in the example
  - $\mathcal{N}(\mathrm{min}, S) = \{\mathtt{Task} \to \mathrm{min}\}$
  - $\mathcal{N}(x, S) = \emptyset$
  - $\mathcal{N}(x, \mathtt{Expr}) = \{\mathtt{Factor} \to x\}$
  - $\mathcal{N}(\mathtt{Term}, \mathtt{Expr}) = \{\mathtt{Expr} \to \mathtt{Term}, \mathtt{Term} \to \mathtt{Term} * \mathtt{Factor}\}$
- computed by graph walk
- can be cached
  - but must be recomputed if the grammar changes (at least locally)

Operations

- $match_\varepsilon(n), n \in N_0$ ... derive ε from n
  - top-down
  - ignore recursion (would produce ∞ly many parse trees)
- *shift* ... read in the next token
- $reduce(s, z), s \in \Gamma, z \in N$... reduce s to z
  - different from LR *reduce*!
  - already reduce after the first symbol
  - *reduce* must complete the match
- $match(s), s \in \Gamma = N \cup T$
  - if $s \in N_0$: $match_\varepsilon(s)$, remember result
  - $t = shift$
  - if $s \in T$: compare s with t
  - if $s \in N$: $reduce(t, s)$

Reduce(s,z) Step

- pick a rule $c \to n_1\ n_2\ \ldots\ n_k\ s\ \alpha_1\ \alpha_2\ \ldots\ \alpha_\ell$ in $\mathcal{N}(s,z)$
- for each $n_i \in N_0$:
  - $match_\varepsilon(n_i)$
- s already recognized . . . parse tree $\mathcal{S}$
- for each $\alpha_j \in \Gamma = N \cup T$:
  - $match(\alpha_j)$ (top-down step)
- parse tree:



- if $c \neq z$: continue reducing recursively
  - $reduce(c, z)$
- (also consider $reduce(z, z)$ for left recursion)

Analysis

- more conflicts as for LR/GLR
  - reduce already after the 1st symbol
  - no lookahead
- but need neither states nor tables
  - initial graph can be dynamically added to
- implementation keeps efficiency even in case of conflicts
  - never execute the same match step more than once

Additional Features (1/2)

- predictive parsing
  - incremental operation driven by shift steps
  - keep explicit parse stacks
  - can predict next token from stacks
    - top-down expansion
- efficient storage reduces effort
  - DAG-structured stacks
    - $\Rightarrow$ match steps executed only once
  - syntax trees as DAGs (*packed forests*)
    - less storage and storing effort
    - avoids some case distinctions
  - allows efficient exhaustive parsing

Additional Features (2/2)

- constraints enforced during parsing
  - parallel multiple context-free grammars (PMCFGs, Seki et al.)
    - parsed as constrained CFGs
  - next token constraints
    - require (expect) / forbid (taboo) token(s) after a rule
    - special case: scannerless parsing (token = character), e.g. maximal matches
- CFG rules can have labels
  - reproduced in the parse tree
- custom parse actions
  - algorithm as presented generates only parse tree
  - want e.g., action for definition producing rule

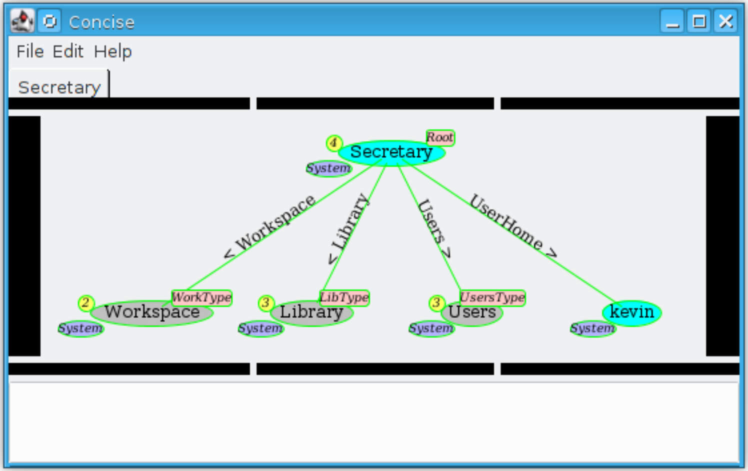
Interoperability Features

- arbitrary token source (lexer, token buffer, . . .)
  - other DynGenPar instance (hierarchical parsing)
    - or other arbitrary parser
  - can also return whole parse tree
  - Flex lexer (C++)
  - character token source (scannerless parsing)
- import of PGF files from Grammatical Framework (GF)
  - see Applications section
- Java bindings
  - built using Qt Jambi generator
  - used in the Concise GUI (see next slide)

Concise

- graphical user interface (GUI)
- integrated development environment (IDE) for FMathL
- implements and visualizes a **semantic memory**
  - graph-structured knowledge representation
- interactive graph editor
- written in Java
- DynGenPar integrated into Concise
  - through the Java bindings
  - used for almost all parsing tasks in Concise

Screenshot of Concise (Graph View)



Embedding of DynGenPar into Concise

- **type sheet**: text file representing a **type system**
  - specification of types
    - required fields
    - optional fields
  - **usages** encoding grammar information
- Concise `Grammar` Java class
  - converts Concise type sheet to DynGenPar grammar
  - converts DynGenPar parse tree to Concise record
    - **record**: subgraph in the semantic memory
- supported token sources:
  - scannerless `TextByteTokenSource`
  - `ConciseTokenSource` producing tokens from a record
    - used to convert list-of-words record to semantic record

Concise Type Sheets

- representation of grammars in Concise
- themselves parsed using DynGenPar
- grammar is itself a type sheet
  - originally by Arnold Neumaier
- bootstrap parser
  - grammar manually converted to a C++ program
  - uses DynGenPar directly (without Concise)
  - produces Concise record as a text file
- can self host (passes bootstrap comparison)
  - can parse type sheets using the bootstrapped type sheet for type sheets
  - produced output matches bootstrap parser

Concise Code Sheets

- fundamental programming language in Concise
- textual representation of **elementary acts**
  - primitive operations on the semantic memory
  - 16 act types: `Do`, `Return`, `Goto`, `Assign`, `Set`, `Get`, `GetType`, `IsSubtypeOf`, `Identical`, `Convert`, `Vcopy`, `ForAllFields`, `Call`, `Ask`, `Supervise`, `Resume`
  - more structured than assembly: loops, etc.
- code sheet representation adds **declarations**
  - types, global and local variables, etc.
  - DynGenPar produces references by name
  - must be **resolved** when converting to elementary acts

Concise Record Transformation Sheets

- transform one record into another
  - usually of a different type
  - fixed **source type** and **destination type**
  - can be the same type (simplifier)
- structured like XSLT style sheets
- adapted to the Concise semantic memory
- added support for name resolution
- used, e.g., for the conversion from code sheets to elementary acts

Chemical Process Modeling (`ChemProcMod`)

- idea, input from Ali Baharev and Arnold Neumaier
  - optimization techniques for chemical process simulation
- attempted using Modelica
  - unsatisfactory syntax
  - issues with available implementations
    - bugs, performance
- specialized modeling language (Concise type sheet)
  - intuitive for a chemical engineer
    - Ali Baharev has chemical engineering background
  - declarative rather than imperative
  - vocabulary from the chemical application
    - not from the mathematical model or OOP
    - e.g., Modelica *class* → ChemProcMod **unit**
- comes with basic unit library

Extensible `OptProbl` Grammar

- context: COCONUT Project (Hermann Schichl et al.)
  - framework for global optimization
- need for extensible input format
  - e.g., adding Lie group notation
- PoC grammar for optimization problems
  - coded directly in C++ using DynGenPar (no Concise)
  - small subset of LaTeX
    - only programming language notation (e.g., $2 * x$)
    - e.g., no implied multiplication (e.g., $2x$)
  - extensible with `\newcommand`
    - adds rules to the grammar at runtime
  - also features next token constraints
    - used to determine the end of a tag

Robust AMPL

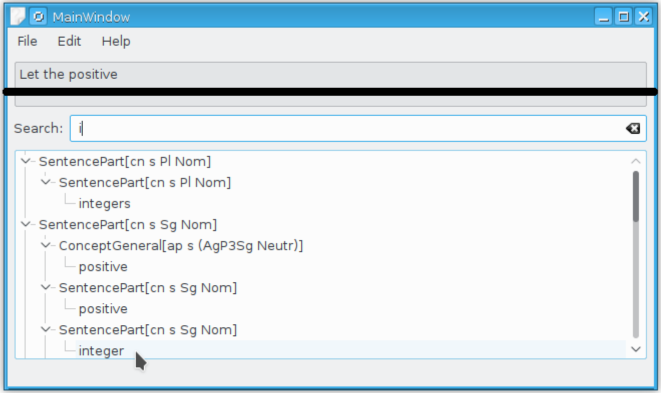
- **AMPL**: A Mathematical Programming Language
  - well-known modeling language for optimization problems
- subset reimplemented as Concise type sheet
  - official AMPL software not needed
- **robust**: added full support for intervals
  - official AMPL: intervals allowed only as sets
  - robust AMPL: intervals allowed wherever numbers are
- record transformation to internal representation
  - to `RobustOptProb` type sheet (Ferenc Domes)
  - record transformation can be **rigorous** (all numbers become intervals with outward rounding) or not (output numbers are of `double` type)

PGF (Portable Grammar Format) File Import

- GF = Grammatical Framework (Ranta et al.)
- PGF (Angelov et al.): format produced by GF
  - based on parallel multiple context-free grammars (PMCFGs, Seki et al.)
  - binary serialization of Haskell data structures
- import implemented in DynGenPar
  - Haskell-compatible deserialization
  - conversion to standard PMCFG format
    - some needed extensions (context-free categories as tokens, next token constraints, tokens with values) supported by DynGenPar
- parsing requires GF-compatible lexer
  - implemented as `PgfTokenSource`

PGF GUI – Graphical Demo Application

demonstrates DynGenPar PGF import and prediction



GF Application Grammar for Mathematical Language

- joint work with Peter Schodl
- early research on grammars for natural mathematical language in the FMathL project
- focus on **linearization** (the opposite of parsing)
  - using GF Haskell runtime (no DynGenPar)
- only for text, formulas as verbatim strings
- 2 versions
  1. handwritten GF grammar
     - in lockstep with the Concise type system
  2. automatically generated GF grammar
     - generated from the Concise type system
- abandoned in favor of Concise type sheets

Naproche (Cramer et al.)

- controlled natural language for mathematical logic
- grammar described in Kühlwein's diploma thesis
- implemented by me
  1. in Bison, using Generalized LR (GLR)
  2. in DynGenPar (in C++), for comparison
- hierarchical grammar
  - formulas parsed by separate grammar
- all 4 grammars use a Flex lexer
  - Bison: Flex C mode
  - DynGenPar: Flex C++ mode (C++ classes)

TextDocument Toolchain

- imports LaTeX document as Concise record
  - in the TextDocument type system

1. LaTeXML (3rd party): converts LaTeX to XML
2. processxml: transforms LaTeXML XML to record XML
3. xmltocnr: converts record XML to Concise record sheet

- central tool: processxml
  - transforms LaTeX structure to TextDocument structure
- resulting TextDocument records are unparsed
  - only document structure represented
  - paragraph = list of words
  - can be parsed in a later step

BasicDefinitions

- Concise type sheet operating on paragraphs
  - unparsed `TextDocument` representation (list of words)
  - uses the `ConciseTokenSource`
- by Arnold Neumaier, Ferenc Domes, Kevin Kofler
- PoC for the handling of mathematical definitions
- main feature: definitions automatically trigger a hook that dynamically adds a rule at runtime
- static part covers just enough idiomatic mathematical English to represent basic mathematical definitions
- formulas kept as unparsed strings
  - can be parsed with separate formula grammar

BasicReasoning

- Concise type sheet operating on paragraphs
- by Arnold Neumaier and Kevin Kofler
- based on `MathNat` by Muhammad Humayoun
- idiomatic mathematical English needed to represent basic mathematical reasoning
  - significantly broader scope than `BasicDefinitions`
- work in progress
  - grammar not yet complete

LaTeX Formulas

- Concise type sheet (by me) operating on strings
- grammar for LaTeX formulas in typical notation
  - e.g., implied multiplication allowed
- by design, not all ambiguities resolved during parsing
  - some resolvable by semantic analysis (planned)
  - remaining ones must be resolved interactively
- scannerless grammar (character tokens)
- based on the expression grammar pattern
- tested on 2 university text books in German
  - $> 70\%$ success rate (see Results section)

Competitive with Bison

- benchmarking results on the Naproche grammar

	compilation	grammar conversion	parsing (Burali-Forti)
Bison	1089 ms	153 ms*	1.60 ms
DynGenPar	8851 ms	5.34 ms	9.37 ms**

* . . . at compile time, thus requires recompilation
** . . . total execution time of 14.71 ms minus grammar conversion time

- only 6 times slower than Bison at pure parsing
  - i.e. exactly what Bison is optimized for
- 29 times faster than Bison at grammar conversion
  - no recompilation required
  - $\Rightarrow$ effectively over 200 times faster!

Competitive with GF (Grammatical Framework)

- benchmarking results on the GF *Phrasebook* grammar
  - testcase:
    See you in the best Italian restaurant tomorrow!

	parsing time
GF Haskell runtime	43.4 ms
GF C runtime	17.8 ms
DynGenPar	121.8 ms

- parsing time comparable to both GF runtimes
  - on practical application grammars
- DynGenPar honors next token constraints
  - both GF runtimes incorrectly accepted
    *Where is a**n** restaurant?*

Performance of Dynamic Rule Addition

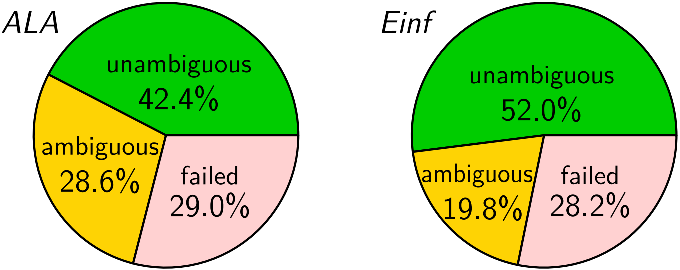
- test results on `BasicDefinitions` grammar
  - paragraphs containing 1 definition each

number of definitions	parsing time
0	7 ms
1	9 ms
10	31 ms
100	228 ms
1000	2298 ms

- time to parse a paragraph and add a rule to the grammar: only 2.3 ms altogether

LaTeX Formula Parser Success Rates

- unique formulas from 2 university textbooks
  - *ALA* (Arnold Neumaier: *Analysis und lineare Algebra*)
  - *Einf* (Hermann Schichl and Roland Steinbauer: *Einführung in das mathematische Arbeiten*)
  - multiple instances of the exact same formula deleted



Possible Future Extensions

- context-sensitive constraints on rules
  - generalize PMCFG and next token constraint support
  - main objective: figure out the needed class of constraints
- stateful parse actions (more state information)
- runtime parser for rules (directly in DynGenPar)
  - read rules into parser from user-writable format
  - allows dynamic extension by the user at runtime
  - now only possible through Concise
- scalability to larger PMCFGs
  - generalize optimizations that assume no constraints
- error correction (long-term research goal)
  - have only basic error detection and reporting
  - goal: suggest corrections to the user

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