1. Compute the characteristic polynomial of
   (a) the matrix $\begin{pmatrix} 0 & 0 & 3 \\ 0 & 4 & 0 \\ 3 & 0 & 8 \end{pmatrix}$, and
   (b) the matrix $\begin{pmatrix} 4 & 1 & 0 \\ -2 & 0 & -2 \\ 0 & 1 & 4 \end{pmatrix}$.

2. Compute an eigenvalue decomposition (spectral decomposition) of the matrix

$$\begin{pmatrix} 0 & 0 & 3 \\ 0 & 4 & 0 \\ 3 & 0 & 8 \end{pmatrix}$$

   from example 1 (a). Then, produce (without redoing computations) a Jordan decomposition and a Schur decomposition of this matrix. Why is it not necessary to compute a new decomposition even for the Schur decomposition in this example?

3. Compute a Jordan decomposition of the matrix

$$\begin{pmatrix} 4 & 1 & 0 \\ -2 & 0 & -2 \\ 0 & 1 & 4 \end{pmatrix}$$

   from example 1 (b). Does this matrix have an eigenvalue decomposition? Why?

4. Compute a Schur decomposition of the matrix

$$\begin{pmatrix} 4 & 1 & 0 \\ -2 & 0 & -2 \\ 0 & 1 & 4 \end{pmatrix}$$

   from examples 1 (b) and 3.

   Hint: Proceed as in the existence proof from the lecture or the transcript. Begin with $\lambda = 4$ for the computation to stay as simple as possible.

5. We consider the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1+\varepsilon \end{pmatrix}$$

   (a) Compute a Jordan decomposition of A for $\varepsilon > 0$. What happens to the decomposition when ε goes to 0? What is the Jordan decomposition for $\varepsilon = 0$?
   (b) What is the Schur decomposition of A?
   (c) Which of the two decompositions is more stable? Why?

6. Determine the eigenvalues of the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Does this matrix have:
   (a) a real-valued eigenvalue decomposition?
   (b) a real-valued Jordan decomposition?
   (c) a real-valued Schur decomposition?

   Justify your answer.

7. (a) Write a program (a function) in MATLAB or Octave which performs the power iteration on any matrix with any starting vector (i.e. the matrix and the starting vector should be arguments of the function).
   (b) Apply the program to the matrix
   $$\begin{pmatrix} 13 & 4 & 3 & 9 \\ -1 & -8 & 5 & 0 \\ 2 & 3 & 7 & 1 \\ 6 & -2 & 0 & 4 \end{pmatrix}$$
   with the starting vector $\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}^\mathsf{T}$. What does the iteration converge to? Hint: Do not expect “nice” numbers as results in this example.

8. Apply the power iteration to the matrix
   $$\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$
   with multiple different starting vectors. What happens to the convergence? Why?

9. Apply the power iteration to the matrix
   $$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$
   with the starting vector $\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^\mathsf{T}$. (You should not need a computer program for that!) Does the algorithm converge on this matrix? Why?